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ON SINGULARITIES OF THE STATE OF STRESS OF AN ORTHOTROPIC HALF-STRIP*

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The state of stress is investigated in a half-strip on whose lateral sides an arbitrary load is given, while boundary conditions of the first or second kind are realized on the endface. The method of Fourier integral transforms is used which enables the problem to be reduced to a singular integral equation with moving and fixed singularities. It is shown that in the case of a clamped endface the stresses have singularities at the angular points of the half-strip, and the dependences of the degree of the singularity on the elastic characteristics of the material are also determined. Graphs of the stress distribution along the clamped endface are constructed for different kinds of external load.

Investigations of the plane problem of elasticity theory for an isotropic half-strip by the method of integral equations, that take account of the stress behaviour in the neighbourhood of the angular points were carried out in /1-3/. A solution /1/ has been constructed for the case of a symmetric load acting on the lateral sides of the clamped half-strip. The case of a load applied infinitely remotely from the endface is examined in /2, 3/.

1. We consider an orthotropic half-strip $0 \leqslant x_1 < \infty$, $|x_2| \leqslant h$ on whose sides an arbitrary stress distribution is given, which without loss of generality we can represent in the form

$$\sigma_{12}(x_1, \pm h) = \begin{Bmatrix} \pm P_1(x_1) \\ P_1(x_1) \end{Bmatrix}, \quad \sigma_{22}(x_1, \pm h) = \begin{Bmatrix} P_2(x_1) \\ \pm P_2(x_1) \end{Bmatrix} \tag{1.1}$$

while on the endface there is one of the following versions of the boundary conditions:

$$u_1(0, x_2) = 0, \quad u_2(0, x_2) = 0 \tag{1.2}$$

$$\sigma_{11}(0, x_2) = P(x_2), \quad \sigma_{12}(0, x_2) = 0 \tag{1.3}$$

Here and henceforth the upper expression in the braces corresponds to the symmetric case, and the lower expression to the antisymmetric case. It is assumed that in the case of the boundary conditions (1.3), the system of stresses (1.1), (1.3) is selfequilibrated, and the function $P(x_2)$ for the symmetric problem is even, and for the antisymmetric is odd.

We use the following Hooke's law relationship and the appropriate Lamé equations /4/:

$$\sigma_{ii} = A_{ii}\frac{\partial u_i}{\partial x_i} + A_{12}\frac{\partial u_{3-i}}{\partial x_{3-i}}, \quad \sigma_{12} = A_{66}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) \tag{1.4}$$

$$b_{ii}\frac{\partial^2 u_i}{\partial x_i^2} + \frac{\partial^2 u_i}{\partial x_{3-i}^2} + b_0\frac{\partial^2 u_{3-i}}{\partial x_1\,\partial x_2} = 0, \quad b_0 = b_{12} + 1, \quad b_{ii} = \frac{A_{ii}}{A_{66}} \tag{1.5}$$
$$b_{12} = \frac{A_{12}}{A_{66}}$$

($i = 1, 2$; $A_{11}, A_{12}, A_{22}, A_{66}$ are stiffness characteristics of the material).

Applying the Fourier integral transform /5/ to the Eqs.(1.5)

$$\bar u_1(t, x_2) = \int_0^\infty u_1(x_1, x_2)\sin tx_1\,dx_1 \quad \bar u_2(t, x_2) = \int_0^\infty u_2(x_1, x_2)\cos tx_1\,dx_1$$

we obtain a system of equations in the transforms

$$\bar L_1 \equiv \frac{d^2\bar u_1}{dx_2^2} - tb_0\frac{d\bar u_2}{dx_2} - t^2 b_{11}\bar u_1 = -tb_{11}u_1(0, x_2) \tag{1.6}$$
$$\bar L_2 \equiv b_{22}\frac{d^2\bar u_2}{dx_2^2} + tb_0\frac{d\bar u_1}{dx_2} - t^2\bar u_2 = b_{12}\frac{\partial u_1}{\partial x_2}(0, x_2) + A_{66}^{-1}\sigma_{12}(0, x_2)$$

For the boundary conditions (1.2), (1.3) under consideration, the right sides of system (6) can obviously not be calculated explicitly. Consequently, introducing the functions

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$U(x_2)$, $q(x_2)$ and $\bar{U}(p)$, $\bar{q}(p)$ by means of the formulas

$$\int_0^\infty \bar{U}(p)\chi_1(px_2)\,dp = U(x_2) = \begin{cases} u_1(0, x_2), & |x_2| \leqslant h \\ 0, & |x_2| > h \end{cases} \tag{1.7}$$

$$\int_0^\infty \bar{q}(p)\chi_2(px_2)\,dp = q(x_2) = \begin{cases} \sigma_{12}(0, x_2), & |x_2| \leqslant h \\ 0, & |x_2| > h \end{cases} \tag{1.8}$$

$$\chi_1(x) = \begin{Bmatrix} \cos(x) \\ \sin(x) \end{Bmatrix}, \quad \chi_2(x) = \begin{Bmatrix} \sin(x) \\ \cos(x) \end{Bmatrix}$$

we write system (1.6) in the form

$$\bar{L}_1 = -tb_{11}\int_0^\infty \bar{U}(p)\chi_1(px_2)\,dp \tag{1.9}$$

$$\bar{L}_2 = -\int_0^\infty [b_{12}p\bar{U}(p) - A_{66}^{-1}\bar{q}(p)]\chi_2(px_2)\,dp; \quad |x_2| \leqslant h$$

The form of the solution of system (1.9) depends on the roots of the characteristic equation

$$k^4 - 2\alpha_1^2k^2 + \alpha_2^4 = 0 \tag{1.10}$$

$$\alpha_1^2 = [b_{11} - (2 + \nu_2)b_{12}]/2, \quad \alpha_2^4 = A_{11}A_{22}^{-1}, \quad \nu_i = A_{12}A_{ii}^{-1}$$

The following cases are therefore possible: 1) $\alpha_1^2 > \alpha_2^2$ are real roots; 2) $\alpha_1^4 < \alpha_2^4$ are complex roots; 3) $|\alpha_1^2| > \alpha_2^2$, $\alpha_1^2 < 0$ are pure imaginary roots. Taking into account that case 2) is encountered comparatively rarely in practice, while 3) is practically not encountered generally, we investigate material of type 1 in an example by noting only the fundamental results for a material of type 2.

Determining the solution of system (1.9) satisfying the transforms of the boundary conditions (1.1), applying the inversion formulas in x_1 /5/, and taking account of the relations

$$\bar{U}(p) = \frac{2}{\pi}\int_0^\infty U(y)\chi_1(py)\,dy, \quad \bar{q}_2(p) = \frac{2}{\pi}\int_0^\infty q(y)\chi_2(py)\,dy \tag{1.11}$$

we obtain the following expressions for the deformation and stress which are needed in the subsequent analysis

$$u_2'(0, x_2) = \frac{\gamma_1}{2\pi}\int_{-h}^{h}\frac{U'(y)}{x_2 - y}\,dy + \frac{\gamma_2}{2\pi A_{22}}\int_{-h}^{h}\frac{q(y)}{x_2 - y}\,dy + \Omega_1(x_2) \tag{1.12}$$

$$\sigma_{11}(0, x_2) = \frac{A_{11}\gamma_3}{\pi}\int_{-h}^{h}\frac{U'(y)}{x_2 - y}\,dy - \frac{\gamma_4}{\pi}\int_{-h}^{h}\frac{q(y)}{x_2 - y}\,dy + \Omega_2(x_2) \tag{1.13}$$

$$\gamma_1 = (\alpha_4 b_{22})^{-1}\lambda_3, \quad \gamma_2 = \alpha_4^{-1}(1 + b_{21}), \quad \gamma_3 = (2\alpha_4 b_{11})^{-1}\lambda_1, \quad \gamma_4 = -\gamma_1/2$$

$$\Omega_i(x_2) = r_i\int_0^\infty \Delta_t^{-1}[w_{2i-1}(x_2, t)I_1(t) + w_{2i}(x_2, t)I_2(t)]\,dt$$

$$r_1 = 2\pi^{-1}, \quad r_2 = r_1A_{11}, \quad \Delta_t = S_1S_4X_3(t) - S_2S_3X_4(t)$$

$$w_{2i-1}(x_2, t) = c_{2i-1,1}X_1(x_2, t) - c_{2i-1,2}X_2(x_2, t)$$

$$w_{2i}(x_2, t) = c_{2i,2}X_3(t)X_2(x_2, t) - c_{2i,1}X_4(t)X_1(x_2, t)$$

$$S_i = \delta_ik_i - 1, \quad S_{2+i} = k_i + \nu_2\delta_i, \quad c_{11} = S_4k_1, \quad c_{12} = S_3k_2$$

$$c_{21} = S_2k_1, \quad c_{22} = S_1k_2, \quad c_{31} = S_4\delta_3, \quad c_{32} = S_3\delta_4, \quad c_{41} = S_2\delta_3, \quad c_{42} = S_1\delta_4$$

$$\delta_i = (1 - b_{22}k_i^2)(b_0k_i)^{-1}, \quad \delta_{2+i} = \delta_i + \nu_1k_i, \quad k_i = \alpha_4 + (-1)^i\alpha_5$$

$$\alpha_3 = 2\alpha_4\alpha_5, \quad \alpha_{i+3} = \sqrt{[\alpha_1^2 - (-1)^i\alpha_2^2]/2}, \quad b_{21} = \sqrt{b_{11}b_{22}}$$

$$X_i(x_2, t) = \chi_3(t_ix_2)/\chi_3(t_ih), \quad X_{i+2}(t) = \chi_4(t_ih)/\chi_3(t_ih)$$

$$\chi_3(x) = \begin{Bmatrix} \operatorname{ch} x \\ \operatorname{sh} x \end{Bmatrix}, \quad \chi_4(x) = \begin{Bmatrix} \operatorname{sh} x \\ \operatorname{ch} x \end{Bmatrix}, \quad t_i = k_it$$

$$I_i(t) = I_i^U(t) + I_i^q(t) + \bar{P}_i(t)$$

$$I_1^U(t) = \frac{\lambda_1}{4}\int_{-h}^{h} U'(y)[\alpha_4^{-1}V_1(y, t) - \alpha_5^{-1}V_2(y, t)]\,dy$$

$$I_2{}^U(t) = -(2\alpha_3)^{-1}\lambda_2 \int_{-h}^{h} U'(y) V_2(y,t)\,dy$$

$$I_1{}^q(t) = (2A_{22})^{-1} \int_{-h}^{h} q(y)[V_1(y,t) m_1 - V_2(y,t) m_2]\,dy$$

$$I_2{}^q(t) = -(2A_{22})^{-1} \int_{-h}^{h} q(y)[V_1(y,t) - V_2(y,t) m_3]\,dy$$

$$\bar{P}_1(t) = A_{66}^{-1}\int_0^\infty P_1(x_1)\sin tx_1\,dx_1, \quad \bar{P}_2(t) = A_{22}^{-1}\int_0^\infty P_2(x_1)\cos tx_1\,dx_1$$

$$m_1 = \lambda_3(2\alpha_4)^{-1}, \quad m_2 = (b_{12} + b_{21})(2\alpha_5)^{-1}, \quad m_3 = \lambda_1(2\alpha_3)^{-1}$$

$$\lambda_1 = \nu_2 b_{12} - b_{11}, \quad \lambda_2 = \lambda_1 b_{22}{}^{-1}, \quad \lambda_3 = b_{12} - b_{21}$$

$$V_i(y,t) = \exp[-\alpha_4(h-y)t]\,\chi_{i+2}[\alpha_5(h-y)t]$$

Relations (1.12) and (1.13) have been obtained taking the following integration formulas into account:

$$\int_0^\infty p\bar{U}(p)\chi_1(px_2)\,dp = \frac{1}{\pi}\int_{-h}^{h} \frac{U'(y)}{x_2 - y}\,dy \tag{1.14}$$

$$\int_0^\infty \bar{q}(p)\chi_2(px_2)\,dp = -\frac{1}{\pi}\int_{-h}^{h} \frac{q(y)}{x_2 - y}\,dy$$

whose validity follows from (1.11) and the limit value of the integral /6/

$$\lim_{z\to 0}\int_0^\infty \exp(-pz)\sin p(y - x_2)\,dp = (y - x_2)^{-1}$$

2. We assume that the endface of the half-strip is rigidly clamped. Then from the first relationship of (1.2) it follows that $U(x_2) = 0$, $\bar{U}(p) = 0$, $I_1{}^U(t) = 0$, $I_2{}^U(t) = 0$, while the second reduces to a singular integral equation in $q(y)$

$$\int_{-h}^{h}\left[\frac{1}{x_2 - y} + M_1{}^\circ(x_2, y)\right] q(y)\,dy = R_1{}^\circ(x_2) \tag{2.1}$$

$$M_1{}^\circ(x_2, y) = \int_0^\infty \Delta_t^{-1} M_1{}^*(x_2, y, t)\,dt$$

$$R_1{}^\circ(x_2) = -4A_{22}\gamma_2^{-1}\int_0^\infty \Delta_t^{-1} R_n{}^*(x_2, t)\,dt$$

$$M_n{}^*(x_2, y, t) = Z_{2n-1}(x_2, t)\exp[-k_2(h-y)t] + Z_{2n}(x_2, t)\exp[-k_1(h-y)t] \tag{2.2}$$

$$R_n{}^*(x_2, t) = w_{2n-1}(x_2, t)\bar{P}_1(t) + w_{2n}(x_2, t)\bar{P}_2(t) \tag{2.3}$$

$$Z_j(x_2, t) = [l_{j1} - l_{j2}X_4(t)]X_1(x_2, t) - [l_{j3} - l_{j4}X_3(t)]X_2(x_2, t) \tag{2.4}$$

$$l_{j,2i-1} = [m_i + (-1)^j m_2] c_{1j}\gamma_2^{-1}, \quad l_{j,2i} = -[1 + (-1)^j m_3] c_{2j}\gamma_2^{-1}$$

(here $n = 1,\ j = 1, 2$).

The integrand $\Delta_t^{-1}M_1{}^*(x_2, y, t)$ has the singularity t^{-1} as $t \to 0$ in the symmetric case, while this function as well as $\Delta_t^{-1}R_1{}^*(x_2, t)$ have the singularity t^{-2} in the antisymmetric case. Using the equilibrium conditions

$$\int_{-h}^{h} q(y)\,dy = \begin{Bmatrix} 0 \\ 2P_2{}^\circ \end{Bmatrix}, \quad P_2{}^\circ = \int_0^\infty P_2(x_1)\,dx_1 \tag{2.5}$$

this singularity can be eliminated if the following equivalent equation is considered in place of (2.1):

$$\int_{-h}^{h}\left[\frac{1}{x_2 - y} + M_1(x_2, y)\right] q(y)\,dy = R_1(x_2) \tag{2.6}$$

$$M_1(x_2, y) = \int_0^\infty \Delta_t^{-1}[M_1{}^*(x_2, y, t) - M_1^{**}(x_2, y, t)]\,dt$$

$$R_1(x_2) = -4A_{22}\gamma_2^{-1}\int_0^\infty \Delta_1^{-1}[R_1^*(x_2,t) - R_1^{**}(x_2,t)]\,dt$$

$$M_n^{**}(x_2,y,t) = \begin{Bmatrix} l_{2n-1,1} - l_{2n-1,3} + l_{2n,1} - l_{2n,3} \\ x_2h^{-2}[t^{-1}(l_{2n-1} + l_{2n}) + y(\lambda^{-1}l_{2n-1} + l_{2n})] \end{Bmatrix} \tag{2.7}$$

$$l_j = l_{j,4} - \lambda l_{j,2} \tag{2.8}$$

$$R_n^{**}(x_2,t) = \begin{Bmatrix} 0 \\ x_2t^{-1}h^{-2}P_2^\circ\omega_n \end{Bmatrix} \tag{2.9}$$

$$\omega_1 = S_2\lambda - S_1\lambda^{-1}, \quad \lambda = k_1/k_2$$

The expressions for M_n^{**} and R_n^{**} are obtained from the Maclauren series expansions of M_n^* and R_n^* in t and retaining the required number of terms.

As $y \to h$ and $x_2 \to \pm h$ the function $M_1(x_2, y)$ becomes unbounded. To isolate the appropriate fixed singularity in explicit form we represent $M_1(x_2, y)$ in the form $M_1(x_2, y) = M_1^\infty(x_2, y) + M_f(x_2, y)$, where $M_f(x_2, y) \in H$ (H is the set of functions satisfying the Holder condition in each of the variables in the interval $[-h, h]$ /7/), and $M_1^\infty(x_2, y)$ is determined by using the asymptotic properties of $M_1^*(x_2, y, t)$ as $t \to \infty$ in the form

$$M_1^\infty(x_2,y) = -\frac{1}{S_5}\sum_{j=1,2}\left[\frac{d_1}{y-h-\lambda\theta_j(x_2)} + \frac{d_2}{y-h-\theta_j(x_2)} + \frac{d_3}{y-h-\lambda^{-1}\theta_j(x_2)}\right]$$

$$\theta_j(x_2) = h + (-1)^j x_2, \quad S_5 = S_1S_4 - S_2S_3, \quad d_1 = (l_{11} - l_{12})k_2^{-1}$$

$$d_2 = (l_{21} - l_{22})k_1^{-1} - (l_{13} - l_{14})k_2^{-1}, \quad d_3 = (l_{24} - l_{23})k_1^{-1}$$

Under the assumption that the unknown function has an integrable singularity, we seek $q(y)$ in the form

$$q(y) = q^*(y)(h^2 - y^2)^{-\alpha}, \quad q^*(y) \in H, \quad 0 \leqslant \operatorname{Re}(\alpha) < 1 \tag{2.10}$$

Let us consider the pratially holomorphic function

$$\Phi(z) = \frac{1}{\pi}\int_{-h}^{h}\frac{q(y)}{y-z}\,dz$$

and let us use the relationships resulting from results in /7/

$$\Phi[h + e\theta_i(x_2)] = -\frac{q^*(h)}{(2h)^\alpha e^\alpha[\theta_i(x_2)]^\alpha \sin\pi\alpha} + \Phi_i^\circ(x_2), \; x_2 \to (-1)^{i-1}h \tag{2.11}$$

$$\Phi(x_2) = \frac{\operatorname{ctg}\pi\alpha}{(2h)^\alpha}\left[\frac{q^*(-h)}{(h+x_2)^\alpha} - \frac{q^*(h)}{(h-x_2)^\alpha}\right] + \Phi_3^\circ(x_2), \quad x_2 \to \pm h$$

$$|\Phi_j^\circ(z)| \leqslant C_j(z \pm h)^{\alpha_0}, \quad \operatorname{Re}(\alpha_0) < \operatorname{Re}(\alpha), \quad e = \lambda, 1, \lambda^{-1}$$

where C_j ($j = 1, 2, 3$) are real constants.

Taking account of the symmetry properties of the unknown function $q(-y) = \{\mp\}\, q(y)$, we obtain a characteristic equation to determine the degree α of the singularity

$$S_5\cos\pi\alpha + d_1\lambda^{-\alpha} + d_2 + d_3\lambda^\alpha = 0 \tag{2.12}$$

from the condition for the existence of a non-trivial solution of (2.6).

It should be noted that in the case of a material of the type 2 the integral equation to determine $q(y)$ is analogous in structure while the corresponding characteristic equation is representable in the form

$$\cos\pi\alpha + \lambda_5 + \lambda_6\cos(\lambda_4\alpha) + \lambda_7\sin(\lambda_4\alpha) = 0 \tag{2.13}$$

($\lambda_4 = \operatorname{arctg}(\bar{\alpha}_5\alpha_4^{-1})$, $\bar{\alpha}_5 = \sqrt{-\alpha_5^2}$, $\lambda_5, \lambda_6, \lambda_7$ are constants governed by the stiffness characteristics of the material).

Using the L'Hopital rule, it can be shown that if the material characteristics tend to be isotropic then relationships (2.12), (2.13) reduce to the equation (ν is Poisson's ratio)

$$(3 - 4\nu)\cos\pi\alpha + 2(\alpha - 1)^2 - 8\nu^2 + 12\nu - 5 = 0$$

which agrees with the corresponding equation to determine the singularity at the apex of an isotropic wedge /8/.

An investigation of the stress σ_{11} is of interest. Using (1.13), we obtain

$$\sigma_{11}(0,x_2) = \frac{1}{\pi}\int_{-h}^{h}\left[\frac{\gamma_4}{y-x_2} + M_2(x_2,y)\right]q(y)\,dy + R_2(x_2) \tag{2.14}$$

$$M_2(x_2, y) = \int_0^\infty \Delta_t^{-1} [M_2^*(x_2, y, t) - M_2^{**}(x_2, y, t)]\, dt$$

$$R_2(x_2) = 2A_{11}\pi^{-1} \int_0^\infty \Delta_t^{-1} [R_2^*(x_2, t) - R_2^{**}(x_2, t)]\, dt$$

The functions M_2^*, R_2^*, M_2^{**}, R_2^{**} are determined by means of the relationships (2.2), (2.3), (2.7), (2.9) for $n = 2$, while Z_j, l_j are determined by (2.4) and (2.8) for $j = 3, 4$, moreover

$$l_{j,2i-1} = [m_1 + (-1)^j m_2]\, c_{3i}\alpha_2^4/2,\ l_{j,2i} = -[1 + (-1)^j m_3]\, c_{4i}\alpha_2^4/2\ (j = 3,\ 4)$$
$$\omega_2 = S_1\delta_4 k_1^{-1} - S_2\delta_3 k_2^{-1}$$

Following the procedure elucidated, the expression for the singular component $\sigma_{11}(0, x_2)$ can be represented in the form

$$\sigma_{11}^c(0, x_2) = -\psi(\alpha) \frac{q^*(h)}{(2h)^\alpha \sin \pi\alpha} \left[\frac{1}{(h - x_2)^\alpha} (\pm) \frac{1}{(h + x_2)^\alpha}\right] \tag{2.15}$$
$$\psi(\alpha) = \gamma_4 \cos \pi\alpha + d_4\lambda^{-\alpha} + d_5 + d_6\lambda^\alpha,\ d_4 = (l_{32} - l_{31})\, S_5^{-1}k_2^{-1}$$
$$d_5 = S_5^{-1}[(l_{33} - l_{34})\, k_2^{-1} - (l_{41} - l_{43})\, k_1^{-1}]$$
$$d_6 = (l_{43} - l_{44})\, S_5^{-1}k_1^{-1}$$

Defining the stress intensity coefficients by the formulas

$$T_1 = \lim_{x_2 \to h} (2h)^\alpha (h - x_2)^\alpha \sigma_{11}(0, x_2), \quad T_2 = \lim_{x_2 \to h} (2h)^\alpha (h - x_2)^\alpha \sigma_{12}(0, x_2)$$

on the basis of (1.8), (2.10), (2.15), we obtain

$$T_1 = -q^*(h)\psi(\alpha)(\sin \pi\alpha)^{-1},\ T_2 = q^*(h),\ T_3 = -T_2T_1^{-1} = [\psi(\alpha)]^{-1}\sin \pi\alpha \tag{2.16}$$

3. If the boundary conditions (1.3) are given on the endface, then it follows from the second condition $q(x_2) = 0$, $\bar{q}(p) = 0$, $I_1^q(t) = 0$, $I_2^q(t) = 0$, and the first results in a singular integral equation in $U'(y)$

$$\int_{-h}^{h} \left[\frac{1}{x_2 - y} + M_3^\circ(x_2, y)\right] U'(y)\, dy = R_3^\circ(x_2) \tag{3.1}$$

$$M_3^\circ(x_2, y) = \int_0^\infty \Delta_t^{-1} M_3^*(x_2, y, t)\, dt$$

$$R_3^\circ(x_2) = \pi A_{11}^{-1}\gamma_3^{-1}P(x_2) - 2\gamma_3^{-1}\int_0^\infty \Delta_t^{-1}R_2^*(x_2, t)\, dt$$

Here M_3^* is determined by (2.2) for $n = 3$ and Z_j by (2.4) for $j = 5, 6$, where

$$l_{j,2i-1} = [m_4 + (-1)^j m_5]\, c_{3i}\gamma_3^{-1}/4,\ l_{j,2i} = (-1)^j m_6 c_{4i}\gamma_3^{-1}/4\ (j = 5, 6)$$
$$m_4 = \lambda_1\alpha_4^{-1},\ m_5 = \lambda_1\alpha_5^{-1},\ m_6 = 2\lambda_2\alpha_3^{-1}$$

The structure of the kernel of (3.1) is analogous to (2.6). Consequently, by using the representation $U'(y) = U^*(y)(h^2 - y^2)^{-\gamma}(U^* \in H)$ and performing an analogous analysis, we obtain a characteristic equation to determine γ

$$S_5\cos\pi\gamma + [(l_{51} - l_{52})\, \lambda^{-\gamma} - l_{53} + l_{54}]\, k_2^{-1} + [l_{61} - l_{62} - (l_{63} - l_{64})\, \lambda^\gamma]\, k_1^{-1} = 0 \tag{3.2}$$

It should be noted that (3.2) is completely equivalent to the characteristic equation obtained in /9/ for the case of a crack intersecting the boundary of a domain. A numerical analysis executed for different values of the stiffness parameters shows that (3.2) has only a zero root in the interval (0, 1). This means that in the case of the boundary conditions (1.3) the stresses have no singularities at the angular points of the half-strip. Consequently, we shall henceforth not investigate this version of the boundary conditions.

4. A numerical investigation of the solutions obtained in the case of a clamped endface was performed for different values of b_{11}, b_{12}, b_{22}. Results of calculating the degree of singularity α (solid lines), as well as the ratios of the stress intensity coefficients (dashes) for $b_{11} = 10$ and $b_{11} = 28$, respectively, are shown in Figs.1 and 2. The light and dark circles correspond to the boundary points of materials of type 1 and 2. The results obtained indicate that the values of α and T_3 decrease as the stiffness grows in the principal orthotropy directions.

In order to determine the values of the stresses along the clamped endface, we obtain the numerical solution of (2.6) under the additional condition (2.5). To do this we use a method based on the Gauss-Jacobi formula of highest algebraic accuracy /10/, whose generalization to the case of a singular integral is given in /11, 12/. The appropriate system of algebraic equations has the form

$$\sum_{j=1}^{n} A_j \left[\frac{1}{\zeta_m - \tau_j} + hM_1(h\zeta_m, h\tau_j) \right] G(h\tau_j) = R_1(h\zeta_m) \tag{4.1}$$

$$\sum_{j=1}^{n} A_j G(h\tau_j) = 0$$

$$A_j = \frac{2^{1-2\alpha}\Gamma^2(n+1-\alpha)}{n!\,\Gamma(n+1-2\alpha)(1-\tau_j^2)[P_n'^{(-\alpha,-\alpha)}(\tau_j)]^2}, \quad G(h\tau_j) = (1-\tau_j^2)^{\alpha} q(h\tau_j)$$

where $\zeta_m\ (m = 1, 2, \ldots, n-1)$ and $\tau_j\ (j = 1, 2, \ldots, n)$ are the zeros of the Jacobi polynomials $P_{n-1}^{(1-\alpha,1-\alpha)}(\zeta)$ and $P_n^{(-\alpha,-\alpha)}(\tau)$, respectively.

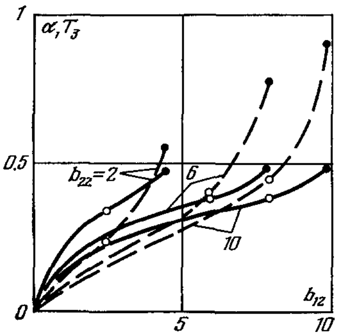

Fig.1

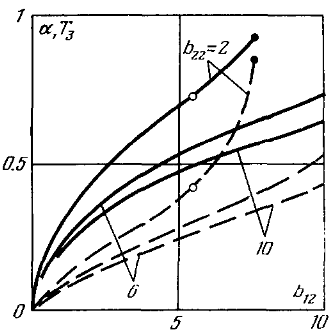

Fig.2

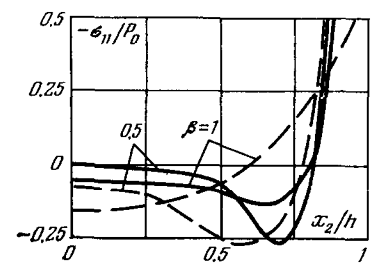

Fig.3

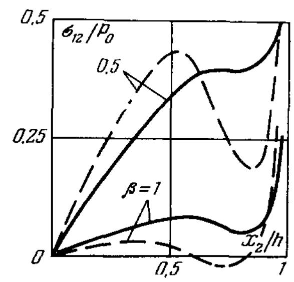

Fig.4

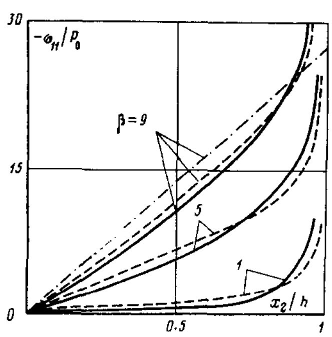

Fig.5

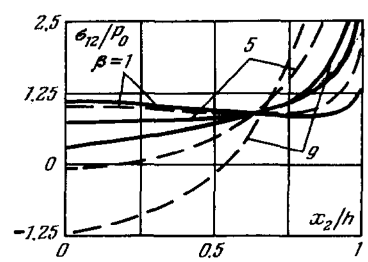

Fig.6

The approximate formula to determine the normal stress is obtained on the basis of (2.14) in the following form

$$\sigma_{11}(0, h\zeta_m) = \frac{1}{\pi} \sum_{j=1}^{n} A_j \left[\frac{\gamma_4}{\tau_j - \zeta_m} + hM_2(h\zeta_m, h\tau_j) \right] G(h\tau_j) + R_2(h\zeta_m) \tag{4.2}$$

The results of calculating the stresses caused by the action of two concentrated forces applied at a distance βh from the endface $(P_1(x_1) = 0,\ P_2(x_1) = P_0\delta(x_1 - \beta))$ are shown in Figs.3 and 4 (symmetric case) and 5 and 6 (antisymmetric). The solid lines are constructed for an orthotropic material with the stiffness characteristics $b_{11} = 26.9,\ b_{22} = 3.6,\ b_{12} = 3.35\ (\alpha = 0.248)$, and the dashes for a quasi-isotropic material for $\nu = 0.3\ (\alpha = 0.295)$. The dash-dot line in Fig.5 corresponds to a beam theory computation of a half-strip. It should be noted that the results shown in Fig.3 for the quasi-isotropic case are in good agreement with the corresponding results in /1/.

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